

The Heat and Moisture Transport Properties of Wet Porous Media¹

B. X. Wang,² Z. H. Fang,^{2,3} and W. P. Yu^{2,4}

Existing methods for determining heat and moisture transport properties in porous media are briefly reviewed, and their merits and deficiencies are discussed. Emphasis is placed on research in developing new transient methods undertaken in China during the recent years. An attempt has been made to relate the coefficients in the heat and mass transfer equations with inherent properties of the liquid and matrix and then to predict these coefficients based on limited measurements.

KEY WORDS: heat transport; mass diffusivity; moisture transport; porous media; thermal diffusivity.

1. INTRODUCTION

The theory of simultaneous heat and mass transfer in porous media has been the essential basis that mankind relies on for its survival and development since ancient times. Therefore, the movement of water in soils and aquifers has been comprehensively studied in soil science [1] and ground water hydrology [2]. Such rational knowledges will be significant also for plant environment [3], agriculture, and environmental protection.

The theory of simultaneous heat and mass transfer in porous media

¹ Invited paper presented at the Tenth Symposium on Thermophysical Properties, June 20–23, 1988, Gaithersburg, Maryland, U.S.A.

² Thermal Engineering Department, Tsinghua University, Beijing 100084, People's Republic of China.

³ Present address: Shandong Insitute of Civil Engineering, Jinan, Shandong Province, People's Republic of China.

⁴ Present address: Department of Power Engineering, Nanjing Institute of Technology, Nanjing, People's Republic of China.

has become of fundamental importance for a multitude of engineering applications. For example, petroleum engineering, which deals with a multicomponent system in porous media, has contributed to the subject [4]. The steam-recovering of petroleum, the exploitation of geothermal energy, and the storage of heat or cold in underground aquifers employ similar theory and techniques. As another example, the energy consumed in heating and air-conditioning of buildings constitutes a considerable fraction of the total energy consumption in the modern society. The characteristics of heat and moisture transfer of building materials affect strongly the economy of operating the building and the comfort of the occupants. In addition, underground structures, which use surrounding soil as a natural insulation, have drawn more attention. So the study on heat and moisture transport in porous media has become an important part of building physics [5].

In modern manufacture, drying is often a major energy-consuming procedure. Thus, the need for improving drying techniques is stimulating the study of heat and moisture transfer in porous media [6]. Moreover, the study of porous media also provides the fundamentals for various branches of science and engineering such as chemical and power engineering, biosphere [7], and space navigation; its applications are still expanding.

Different features and theoretical systems considering the heat and mass transfer in porous media have been formed in respective developments of soil science, petroleum engineering, power and chemical engineering, drying theory, and the theory of heat transfer itself due to their different backgrounds and emphases. Along with interflow and interactions among the different disciplines, however, their approaches and achievements are being mutually adopted. Any breakthrough in the theory of transport process in porous media will promote the progress of all the relevant fields.

It is of vital importance in the study of heat and moisture transfer in porous media to determine all the coefficients in the governing equations or related properties of the medium. This is not only the necessity for establishing and verifying the theory itself, but also an indispensable link between the theory and its application. Nevertheless, there are only a limited number of data on moisture transport properties available so far, and if any, their reliability is usually much poorer than those of heat-transport properties. The heat and moisture transport properties depend strongly upon the temperature, moisture content, and density and structure characteristics of the medium. Besides, determination of the properties is restricted to elementary measurements for temperature and moisture content. In contrast to rather mature techniques for temperature measurement,

adequate approaches are still lacking for quick and reliable determination of the local moisture content in a porous medium, even though quite a number of efforts have been made in this direction as reviewed below.

2. REVIEW OF THE MEASURING TECHNIQUES FOR MOISTURE CONTENT IN POROUS MEDIA

2.1. Loss in Weight

As one of the classical techniques of analysis, the total moisture content of a porous medium can be analyzed by loss in weight on drying a sample at a controlled temperature. Since it employs an absolute measurement, this method has so far been the most reliable and basic one despite the fact that it may also cause the loss of other volatile components. However, in order to determine the moisture content distribution, one has to cut the specimen into pieces. The cutting is sometimes difficult and time-consuming, and especially in the case of sintered materials, it causes unexpected loss in moisture weight. Furthermore, it is a destructive test, so continual monitoring of the change in moisture content is impossible.

2.2. Electrical Capacitance or Conductance [8]

The dielectric constant of water is higher than that of most other materials, and liquid water has a certain electric conductance too. These facts are used to determine the moisture content of porous media. The apparatuses required are relatively inexpensive and rigid, but the effects are dependent on the bulk density, temperature, and chemical composition of the medium. In consequence, although it has been brought into action in some industrial process control, these methods encounter great difficulty in calibration when they are applied to the study of heat and moisture transfer. Also, since the electrode plates are large, the technique is still inadequate for the determination of moisture content distributions.

2.3. γ -Ray Attenuation [9]

Water absorbs γ rays, which can be restricted in a narrow beam of radiation. So this method can be used to determine the moisture content distribution. However, γ -ray attenuation is also affected by the bulk density of the medium, and each measurement takes time to eliminate the fluctuation effect of the radiation; therefore its application in the transient process is limited.

2.4. Microwave Attenuation [10]

Water strongly absorbs microwave radiation at certain frequency bands. Both transmission and reflection of microwave radiation have been used to determine the moisture content of a variety of porous materials, although the former is limited to the application of small thickness and moisture content of the sample. The microwaves pass through the bulk of the medium and hence give a result representing the bulk average of the moisture content on the pass. Physical contact between the sensor and the medium is not necessary. Nevertheless, commercially available equipment using this method cannot yet be focused on a reasonably small area, making this inadequate for measurement of the moisture content distribution. The attenuation is also dependent on the bulk density and composition of the medium and is temperature sensitive as well. These facts make the calibration rather difficult.

2.5. Neutron Moderation [11]

“Fast” neutrons are selectively slowed by protons—the nuclei of hydrogen atoms. Thus the intensity of “slow” neutrons originating from moderation of fast neutrons indicates the concentration of hydrogen atoms in a medium, from which the moisture content is inferred. The neutrons are highly penetrating, so the technique can be used for measurement of thick samples. The measurement is not affected by temperature, pressure, or most common variations in chemical composition but is dependent on the density of the medium. Disadvantages also come from the fairly active radio isotope source it employs. As a result, the equipment is bulky and heavy due to the shielding. Besides, this technique measures the hydrogen rather than the water concentration; therefore hydrogen in the solid or other liquids interferes.

2.6. Low-Resolution Nuclear Magnetic Resonance (NMR) [12]

Like the neutron moderation technique, this method also measures the concentration of hydrogen atoms. As hydrogen nuclei have a magnetic moment, selective resonance absorption of radiation takes place when hydrogen atoms in a magnetic field are exposed to electromagnetic radiation at a certain frequency, which depends on the strength of the magnetic field. Moreover, nuclei in the liquid phase give much sharper absorption maxima than those in solid and, hence, allow determination of the water content in hydrogen-containing solids. According to the relaxation time, i.e., the time for hydrogen atoms to randomize their orien-

tation when released from the magnetic field, one can indicate the form in which water is present. Furthermore, with a gradient of the magnetic field strength introduced, the newly developed NMR imaging technique can detect hydrogen nuclei at a given point (very small space) and then provide a complete picture of the distribution of hydrogen nuclei. This technique has already been used successfully in medicine and, of course, is also a potent means of studying moisture distribution and migration in porous media. It does have limitations though: either metallic or magnetic substances present in the specimen can reduce the accuracy, and temperature compensation is necessary but often difficult. Besides, the equipment is still rather expensive. Also, bonds of the hydrogen nuclei with other atoms in molecules also have a minor influence on the resonance absorption, which has been used to analyze molecular structure by another technique—high resolving-power nuclear magnetic resonance spectroscopy.

2.7. Infrared Reflectance [13]

Water strongly absorbs infrared radiations at certain wavelength bands, at which the solid media are usually opaque to infrared radiation, and the diffuse reflectance from the surface is to be used in the technique; only the moisture content on the surface can thus be detected. The sensor is the noncontacting type and is widely used in industrial control.

2.8. Summary

The direct means for absolute measurement of moisture content according to loss in weight is still the most reliable and fundamental one in the study of heat and moisture transport in porous media, while it suffers from inconvenience of operation and the drawback of destruction of specimens. On the other hand, despite some of their successful applications in process control, all the indirect means of measurement encounter difficulty in calibration, especially when the moisture content varies to a significant extent and changes in temperature must be taken into consideration. The techniques of nuclear magnetic resonance and neutron moderation seem to be promising in this area, but their equipment is still too bulky and expensive to be employed for such limited use at present. Consequently, the lack of adequate means of determining the moisture content in porous media has been one of the main obstacles in the study of heat and moisture transfer in the media and in the determination of moisture transport properties also.

3. DETERMINATION OF THE THERMAL CONDUCTIVITY OF WET POROUS MEDIA

Various methods have been developed for determining experimentally the thermal conductivity of materials under different conditions. These methods may be classified as (i) steady-state methods, (ii) regular thermal regime and quasi-steady methods, and (iii) transient methods. With regard to wet porous media, however, the steady-state methods are seldom employed, for the long test duration of these methods encourages moisture migration.

The regular thermal regime methods [5] refer to a special stage of the transient processes, in which the temperature of a body varies exponentially with time and the temperature profile is independent of its initial distribution. The thermal conductivity is then determined according to the so-called cooling rate of the body. In some particular arrangements, for instance, on constant heating, the temperature profile in the sample remains the same, although the temperature level is still increasing continuously. The situation is referred to as quasi-steady state. A quasi-steady method with a cylindrical geometry was also reported in the use of determining the thermal conductivity of wet soils [14]. An individual experiment based on the normal thermoregime or quasi-steady methods lasts much shorter than steady-state ones, and therefore, the moisture redistribution in the specimen is greatly reduced.

Measurement using transient methods can determine the thermal conductivity and diffusivity simultaneously and takes only minutes or seconds. Therefore, it is more suitable for wet porous materials. One of the transient methods designed for this purpose is the plane heat source method [15], which has been successfully used in determining simultaneously the thermal diffusivity and conductivity of wet soils and building materials [16]. The apparatus is composed of a foil being heated electrically at a constant power. The effect of the heat capacity of the heater on the measurement is proved to be negligible for common building and insulating materials. At the same time, a corrective scheme is presented to compensate this effect in case a thick heater is used or superlight materials are involved [17].

Another transient method, the line heat source method, has been widely used. Being able to determine thermal conductivity only, its testing duration can be as short as less than a minute. Apart from its application in the measurement of liquids and insulation materials, this method has also been employed in wet porous media with the heat capacity effect of the heater discussed [18].

A variant of the line heat source is proposed as thermal probe with heating following by cooling method [19]. The apparatus is composed of

an electrically heated probe which can be inserted into undisturbed soil, and therefore, it is particularly useful for on-site measurement.

4. DETERMINATION OF MOISTURE TRANSPORT PROPERTIES IN POROUS MEDIA

The moisture in wet porous media migrates generally due to the following causes: the total pressure gradient, the moisture content gradient, and the temperature gradient. Under negligible total pressure gradient or in a medium with poor permeability, the moisture migration caused by the latter two causes are prevailing. In such a case, the governing equation of mass transfer takes the following form

$$\frac{\partial W}{\partial \tau} = \nabla \cdot (D_m \nabla w) + \nabla \cdot (D_t \nabla t) \quad (1)$$

where D_m is the "mass diffusivity" and D_t the "thermo-mass diffusivity." The moisture transport properties, D_m and D_t , depend strongly upon the temperature and, especially, the moisture content of the medium. For example, the mass diffusivity of porous media may vary typically through three orders of magnitude from nearly saturated to nearly dry conditions. This makes the governing equation strongly nonlinear. The constant property assumption, which is usually accepted in heat transfer studies, is inappropriate in solving moisture transport problems in general. In addition, the lack of quick and reliable means of measuring moisture content in porous media, as reviewed above, has been a severe obstacle to determining experimentally the moisture transport properties. The capillary hysteresis and possible swelling or shrinking of the medium matrix on wetting or drying further aggravate difficulties in the measurement. In consequence, determination of the moisture transport properties is much more difficult than that of thermal conductivity. Up to now, these data have been very deficient and poor in accuracy.

The methods for determining the moisture transport properties are mainly classified as steady-state methods and transient ones. Due to the difficulties mentioned above, transient methods for determining moisture transport properties are far less mature than those for heat transport properties, and steady-state methods are still prevailing despite the common disadvantage of the rather long duration to establish a steady state.

To determine mass diffusivity, D_m , a one-dimensional steady moisture content distribution is usually maintained in an isothermal specimen, and the moisture flux, J , is measured. Then one ascertains the moisture content distribution in the specimen, from which the moisture content gradient

is evaluated. Thus the mass diffusivity of the specimen is obtained as follows [5]

$$D_m = J / \left(\rho_d \frac{dw}{dx} \right) \quad (2)$$

To determine the thermo-mass diffusivity, D_t , or the "thermogradients coefficient," $\delta = D_t/D_m$, one should try to create a steady temperature gradient as well as a steady moisture content gradient. In the simplest scheme, the specimen is kept moisture-insulated with its ambient. In this case, the moisture flux equals zero everywhere in the specimen, which means the moisture migration caused by the moisture content gradient offsets that by the temperature gradient [14]. That is, from Eq. (1),

$$D_m \frac{dw}{dx} = -D_t \frac{dt}{dx} \quad (3)$$

Then one obtains

$$D_t = -D_m \frac{dw}{dt} \quad (4)$$

or

$$\delta = \frac{D_t}{D_m} = -\frac{dw}{dt} \quad (5)$$

These steady-state methods are based on simple principles and need not assume constant properties in their derivation. In addition, a single experiment can result in several data for the property at different moisture content. However, besides their long test duration, the steady-state methods also suffer from the difficulty in ascertaining the moisture content gradient with a reasonable precision, which is accounted for by (i) insufficiency of potent measuring techniques and (ii) the heterogeneity in structure and porosity which often exist in practical porous materials.

The transient methods for determining moisture transport properties began to develop in the fifties. A few of them were developed by Luikov et al. [5] based on solutions of the moisture content responses under certain boundary and initial conditions. In their solutions, however, the properties were assumed constant in consideration of the complexity in solving non-linear differential equations. The methods of loss in weight and electrical capacitance were used in the measurement of the moisture content. These transient methods have not been widely accepted because of the difficulties and deficiencies mentioned above.

At the same time researchers in soil science concentrated on the isothermal moisture migration and made impressive progress. On this basis Bruce and Klute [21] proposed a transient method for measurement of mass diffusivity in soils. The method measures the moisture content distribution in a specimen undergoing isothermal absorption, and the mass diffusivity is obtained by the equation

$$D_m = -\frac{1}{2} \frac{d\eta}{dw} \int_{w_i}^w \eta dw \tag{6}$$

where $\eta = x\tau^{-1/2}$ is known as Boltzmann’s transform. With the variable property considered, this method can determine the mass diffusivity as a function of the moisture content in a single experiment. Bruce and Klute measured the moisture content distribution by the classical means of loss in weight. Later Gummerson et al. [12] reported their use of nuclear magnetic resonance imaging technique in nondestructive determination of the moisture content distribution. The Bruce and Klute method has to evaluate the moisture content gradient from the measured distribution. Thus the profile of the moisture distribution should be smooth enough, which in fact can hardly be fulfilled. To make up for the deficiency in the measured profile, it is necessary to smooth it with artificially imposed modification. This, of course, brings a considerable uncertainty to the results. Bruce and Klute estimated that the possible error of this method might be 200 to 500% [21].

An exponential relation

$$D_m = D_{m,0} \exp[\beta(w - w_0)] \tag{7}$$

was first suggested by Gardner and Mayhugh [22] to describe the mass diffusivity of soils and later also confirmed on building materials [23]. Being simple in expression, this exponential relation has proved useful in practical applications.

Also based on the isothermal absorption, new methods have been designed by the authors to determine the parameters β and $D_{m,0}$ in Eq. (7) [24, 25]. For this purpose either the cumulative absorptions in two separate test or the cumulative absorption and the absorptive moment in the same test are ascertained. The cumulative absorption, U , and the absorptive moment, V , are defined as

$$U = \int_0^\infty \rho_d(w - w_i) dx \tag{8}$$

$$V = \int_0^\infty \rho_d(w - w_i)x dx \tag{9}$$

In these methods, determination of the moisture content profile and, especially, its gradient is unnecessary; only the overall effect of absorption plays a role. So it is helpful to cope with the obstacle resulting from the possible tortuosity of the profile.

5. DETERMINATION OF HEAT AND MASS TRANSPORT PROPERTIES OF MOIST POROUS MEDIA

A transient method proposed by the authors [26, 27] is based on a more complicated model [28], in which a temperature gradient takes place. The energy equation is expressed as [28]

$$c_p^* \frac{\partial t}{\partial \tau} = \nabla \cdot (\lambda^* \nabla t) + \xi (\nabla t)^2 + \alpha \nabla w \cdot \nabla t \quad (10)$$

where c_p^* denotes the nominal specific heat, λ^* the nominal thermal conductivity, ξ the nonlinear thermal conductivity, and α the mass thermal conductance. With the assumption of constant properties and small temperature and moisture gradients, it was shown [20] that Eq. (10) can be simplified to

$$\frac{\partial t}{\partial \tau} = a_e \nabla^2 t \quad (11)$$

where $a_e = \lambda_e / \rho c_p^*$ is the "effective" thermal diffusivity, and λ_e the effective thermal conductivity, which reflects the overall effect of nominal heat conduction indicated by nominal thermal conductivity, vapor diffusion, and capillary flow of liquid on heat transfer, just the same as that derived in Ref. 20.

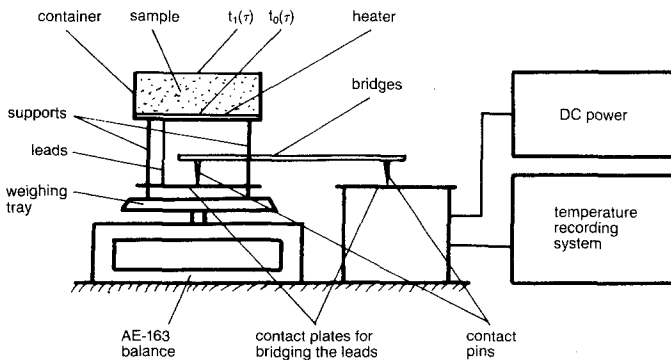


Fig. 1. Schematic diagram of the experimental system.

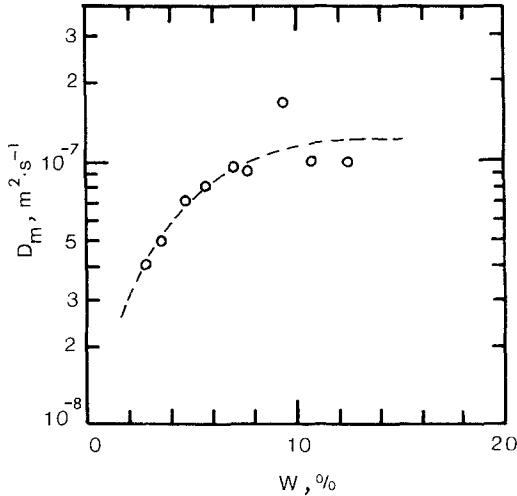


Fig. 2. Mass diffusivity versus moisture content for sand.

The proposed method determines simultaneously the thermal conductivity, thermal diffusivity, mass diffusivity, and thermo-mass diffusivity according to overall effects of heat and moisture transfer in a specimen. The schematic apparatus is shown in Fig. 1. A cylindrical specimen with a uniform initial temperature and moisture content is placed on an electronic balance. It begins to be heated at the bottom with a constant heat

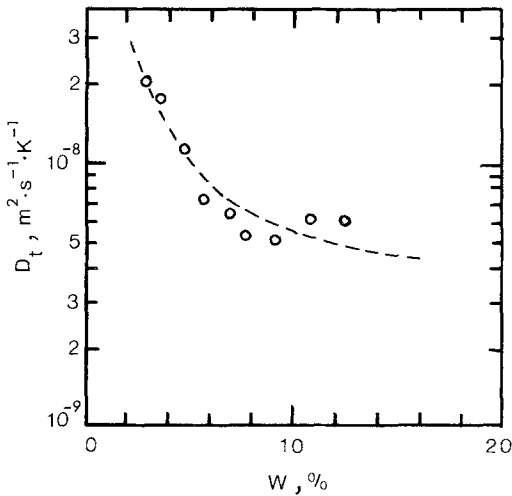


Fig. 3. Thermo-mass diffusivity versus moisture content for sand.

flux while its top surface is opened to the surroundings. The temperature and moisture content responses of the specimen have been obtained analytically under constant property assumption. On the other hand, the temperature responses on both the top and the bottom boundaries are recorded by a highly precise digital balance. Specially designed bridges lead the heater and thermocouples out of the balance. The recorded data on temperature responses and moisture loss are then processed by least-squares fitting to obtain all the four transport properties. A single run of experiment lasts a few minutes and the temperature rise is kept within 3°C .

Because the heat and moisture transport properties of wet porous media are all functions of the temperature and moisture content as well as the density and structural characteristics of the media, it is really an arduous task to determine these properties experimentally with full ranges of change in temperature and moisture content covered. In order to cope with this difficulty, a theory is proposed [29], which reveals intrinsic relations of the properties at different temperatures. Provided that a single set of property data at a reference temperature is determined as a function of the moisture content, the properties at different temperatures can then

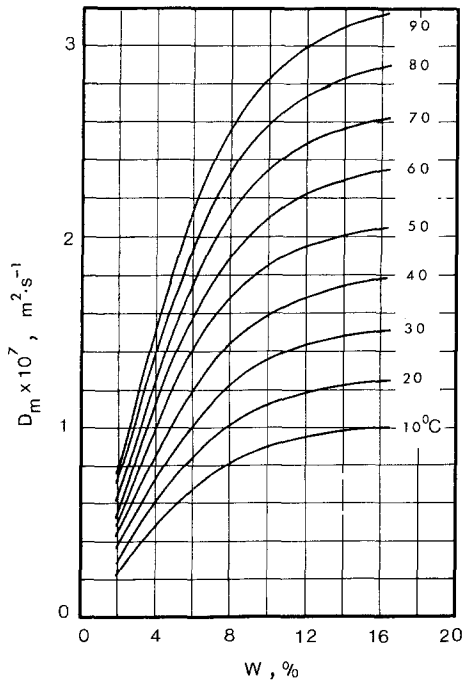


Fig. 4. Predicted mass diffusivity of wet sand.

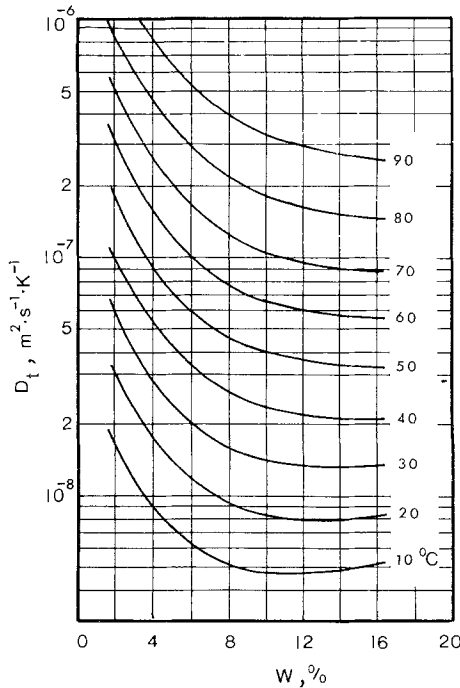


Fig. 5. Predicted thermo-mass diffusivity of wet sand.

be derived from it. In Figs. 2 and 3 examples are shown of the measured mass diffusivity and thermo-mass diffusivity and thermo-mass diffusivity of a fine silica sand, with a grain size diameter of 0.25–0.5 m, $\rho_d = 2510 \text{ kg} \cdot \text{m}^{-3}$, and a porosity of 0.41, at a reference temperature of 20°C [28]. The predicted values at different temperatures from 10 to 90°C are shown in Figs. 4 and 5 [29].

6. CONCLUDING REMARKS

Determination of the heat and moisture transport properties has depended on development of the theory of heat and mass transfer in wet porous media. The experimental measurements of the properties are so far based essentially on simplified theories on which a number of inherent limitations are imposed. The present theory requires, for instance, that the porous medium has to be homogeneous and isotropic in the macroscopic sense and can be regarded as a quasi-continuum so that the usual calculus can be applied to the volume-averaged quantities. It also assumes the medium matrix to be rigid, i.e., it does not swell or shrink during the

processes concerned. The hysteresis in the relation between the capillary potential and the moisture content is a difficult aspect in dealing with moisture migration problems. It is often omitted in usual theoretical models, but attention must be paid to its effect on determination of the moisture transport properties as well as actual moisture migrations. Studies on freezing and thawing in wet porous media are of importance in the construction and maintenance of highways, railways, and buildings. Further theoretical and experimental studies are needed on these phenomena, and so in the determination of the transport properties in these situations.

It should be noted that the methods mentioned in Section 3 determine merely the "nominal" or "apparent" thermal conductivity, which incorporates the contribution of pure conduction with that of the transfer of latent heat by vapor movement and even the transfer of enthalpy by liquid water movement. The pure thermal conductivity should be separated from the apparent thermal conductivity determined in experiments so that heat transfer problems in a wet porous medium can be solved strictly, which requires the solution of the coupled differential equations of heat and mass transfer. Such a separation, however, is difficult by means of heat conduction experiments. A semiempirical approach was employed to obtain the pure thermal conductivity according to Krisher's theory [16]. The approach of the detailed phase average proposed by Whitaker [30] has provided a basis for an insight into the different contributions of the vapor and liquid movements to the total heat flux. However, its practical application is limited due to the considerable number of parameters which should be determined. Very recently, de Vries [31] also presented a valuable discussion on the thermal conductivity in wet porous media.

More difficulties concerning determination of the moisture transport properties are encountered as a consequence of their strong dependency on the moisture content and obstacles in measuring the moisture content. Further research on techniques of determining both the moisture content and the moisture transport properties is still an imminent task.

REFERENCES

1. J. R. Philip, *Adv. Hydrosci.* 5:215 (1969).
2. J. Bear, *Dynamics of Fluids in Porous Media* (American Elsevier, New York, 1972).
3. W. R. van Wijn et al., *Physics of Plant Environment* (North-Holland, Amsterdam, 1963).
4. A. E. Scheidegger, *The Physics of Flow Through Porous Media* (University of Toronto Press, Toronto, 1974).
5. A. V. Luikov, *Heat and Mass Transfer in Capillary-Porous Bodies* (Pergamon Press, Oxford, 1966).
6. R. B. Keey, *Drying Principles and Practice* (Pergamon Press, Oxford, 1972).
7. D. A. de Vries and N. H. Afgan (eds.), *Heat and Mass Transfer in Biosphere* (Scripta, Washington, D.C., 1975).

8. K. Carr-Brion, *Moisture Sensors in Process Control* (Elsevier, London, 1986).
9. F. Q. Huang, in *Nuclear Science and Technology* (Science Press, Beijing, 1964) (in Chinese).
10. A. Kraszewski, *J. Microwave Power* **15**(4):209 (1980).
11. C. H. M. Van Bavel, *Humidity and Moisture 4* (Reinhold, New York, 1965).
12. R. J. Gummerson, C. Hall, and W. D. Hoff, *Nature* **281**:56 (1979).
13. H. A. Willis, *Advances in Infrared and Raman Spectroscopy—2* (Heyden, London, 1976).
14. E. R. G. Eckert and E. Pfender, in *Proceedings of the 6th International Heat Transfer Conference* (1978), Vol. 6, pp. 1–12.
15. B. X. Wang, L. Z. Han, W. C. Wang, and Z. L. Jiao, *Eng. Thermophys. in China* **1**(2):255–268 (Reinhold, New York, 1980).
16. B. X. Wang and R. Wang, *Chin. J. Eng. Thermophys.* **4**:146 (1983).
17. B. X. Wang, L. Z. Han, X. X. Deng, and Z. H. Fang, *Chin. J. Eng. Thermophys.* **4**:38 (1983).
18. B. X. Wang and W. P. Yu, *Chin. J. Eng. Thermophys.* **7**:381 (1986).
19. B. X. Wang and Y. Jiang, *Chin. J. Eng. Thermophys.* **6**:249 (1985).
20. B. X. Wang and Z. H. Fang, *Heat Technol.* **2**:29 (1984).
21. P. R. Bruce and A. Klute, *Soil Sci. Soc. Am. Proc.* **20**:458 (1965).
22. W. R. Gardner and M. S. Mayhugh, *Soil Sci. Soc. Am. Proc.* **22**:197 (1958).
23. C. Hall, W. D. Hoff, and M. Skeldon, *J. Phys. D Appl. Phys.* **16**:1875 (1983).
24. B. X. Wang and Z. H. Fang, *Int. J. Heat Mass Transfer* **31**:251 (1988).
25. Z. H. Fang, Doctoral dissertation (Tsinghua University, Beijing, China, 1987).
26. B. X. Wang, L. Z. Han, and W. P. Yu, *Int. J. Exp. Heat Transfer Thermodyn. Fluid Mech.* **1**:93 (1988).
27. W. P. Yu, Doctoral dissertation (Tsinghua University, Beijing, China, 1987).
28. B. X. Wang and W. P. Yu, in *Heat Transfer Science and Technology* (Hemisphere, New York, 1987).
29. B. X. Wang and W. P. Yu, *Int. J. Heat Mass Transfer* **31**:1005 (1988).
30. S. Whitaker, *Adv. Heat Transfer* **13**:119 (1977).
31. D. A. de Vries, *Int. J. Heat Mass Transfer* **30**:1343 (1987).